Molodensky-Badekas Reversibility (including Dutch Method)

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Topics

- Part I: “Conventional” M-B Reverse by reversing the signs of 7 of the 10 parameters is covered on slides 3 through 13
- Part II: “Dutch” M-B Reverse by reversing the signs of 7 of the 10 parameters and adding the translations to the center of rotation is covered on slides 14 through 23

Part I Contents

- A derivation of the non-linear, higher-order terms of the reversed M-B shift is given on the next slide. Those terms are:

$$R^2 \cdot (u' - u) - R \cdot (2 \cdot \Delta S \cdot (u - u') + \Delta u) + \Delta S^2 \cdot (u' - u) - \Delta S \cdot \Delta u$$

- This presentation offers a "worst-case" computation of the size of these higher-order terms over a very large area
- Non-reversibility is typically only a few centimeters over a large area

Forward Molodensky-Badekas

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} \Delta U \\ \Delta V \\ \Delta W \end{bmatrix} + \begin{bmatrix} 0 & \omega & -\psi \\ -\omega & 0 & \varepsilon \\ \psi & -\varepsilon & 0 \end{bmatrix} \begin{bmatrix} U-U' \\ V-V' \\ W-W' \end{bmatrix}_i + \Delta S \begin{bmatrix} U-U' \\ V-V' \\ W-W' \end{bmatrix}_i$$

$$x = u + \Delta u + R \cdot (u - u') + \Delta S \cdot (u - u')$$

Conventional Reverse M-B

$$u_r = x - \Delta u - R \cdot (x - u') - \Delta S \cdot (x - u')$$

$$u_r = u + [R^2 \cdot (u' - u) - R \cdot (2 \cdot \Delta S \cdot (u - u') + \Delta u) + \Delta S^2 \cdot (u' - u) - \Delta S \cdot \Delta u]$$

Conventional M-B Reverse

- Reversing the M-B transformation means this:
  - Changing the order of the two ellipsoids
  - Negating the signs of 7 of the 10 parameters, viz., ΔX, ΔY, ΔZ, rX, rY, rZ, and ΔScale
  - <u>Maintaining exactly the same geocentric coordinates for the rotation center, viz., X0, Y0 and Z0</u>
- The same rotation center coordinates are used, i.e., only 7 of 10 parameters change (in addition to reversing the order of the ellipsoids)

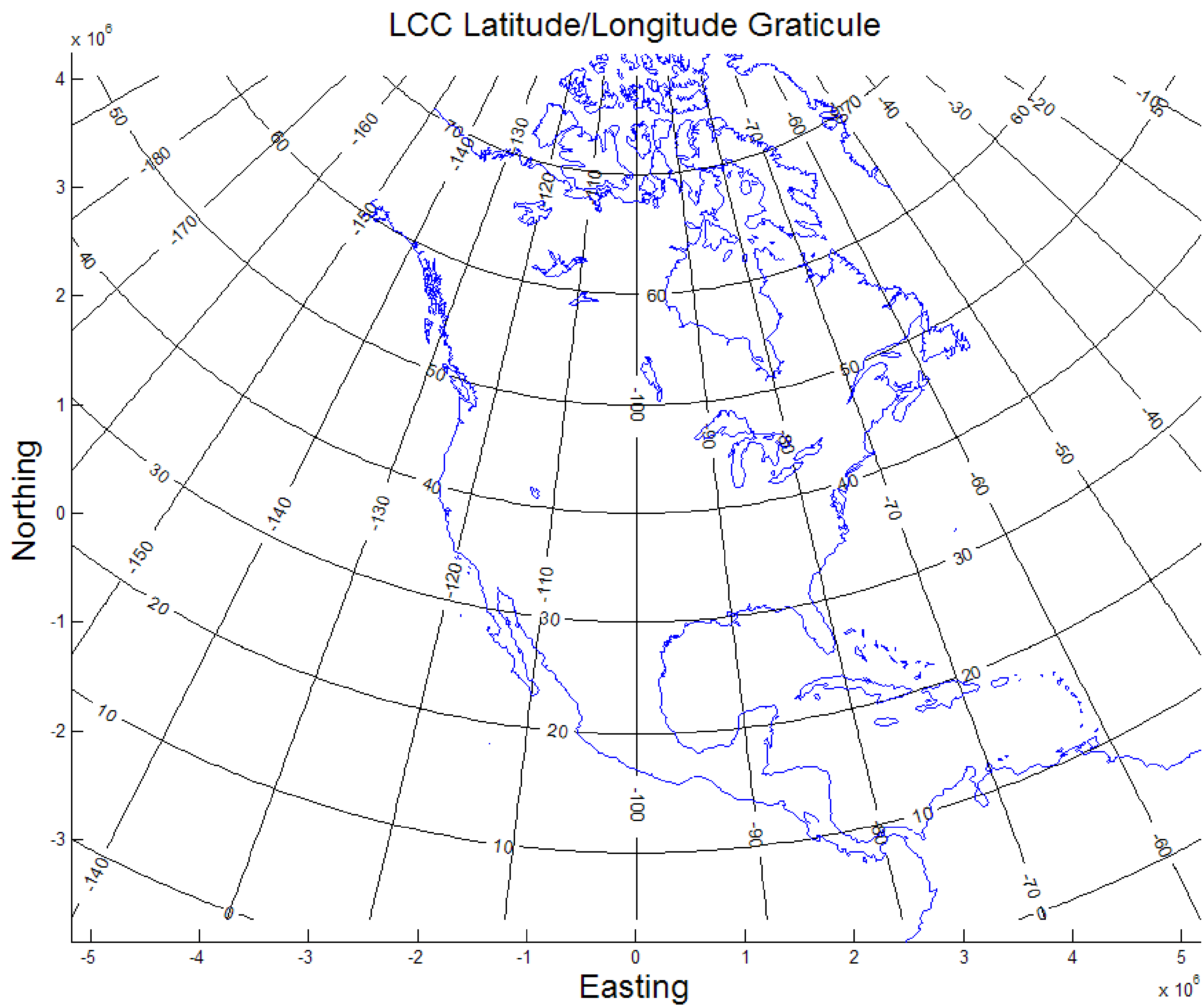
Conventional Forward and Reverse M-B

```
% Forward M-B
Transformation
% Clarke 1866
A_in =      6378206.4;
RF_in =  294.9786982;
dX =             1000;
dY =            -1000;
dZ =             1000;
rotXsec =         -10;
rotYsec =          10;
rotZsec =         -10;
dS =        20*10^-6;
% 40N/100W/0Hgt on Clarke
X0 =      -849632.077;
Y0 =     -4818502.951;
Z0 =      4077787.743;
% Bessel
A_out =   6377397.155;
RF_out = 299.1528128;
```

```
% Reverse M-B
Transformation
% Bessel
A_in =    6377397.155;
RF_in =  299.1528128;
dX =            -1000;
dY =             1000;
dZ =            -1000;
rotXsec =          10;
rotYsec =         -10;
rotZsec =          10;
dS =       -20*10^-6;
% 40N/100W/0Hgt on Clarke
X0 =      -849632.077;
Y0 =     -4818502.951;
Z0 =      4077787.743;
% Clarke 1866
A_out =     6378206.4;
RF_out = 294.9786982;
```

Transformation Parameters

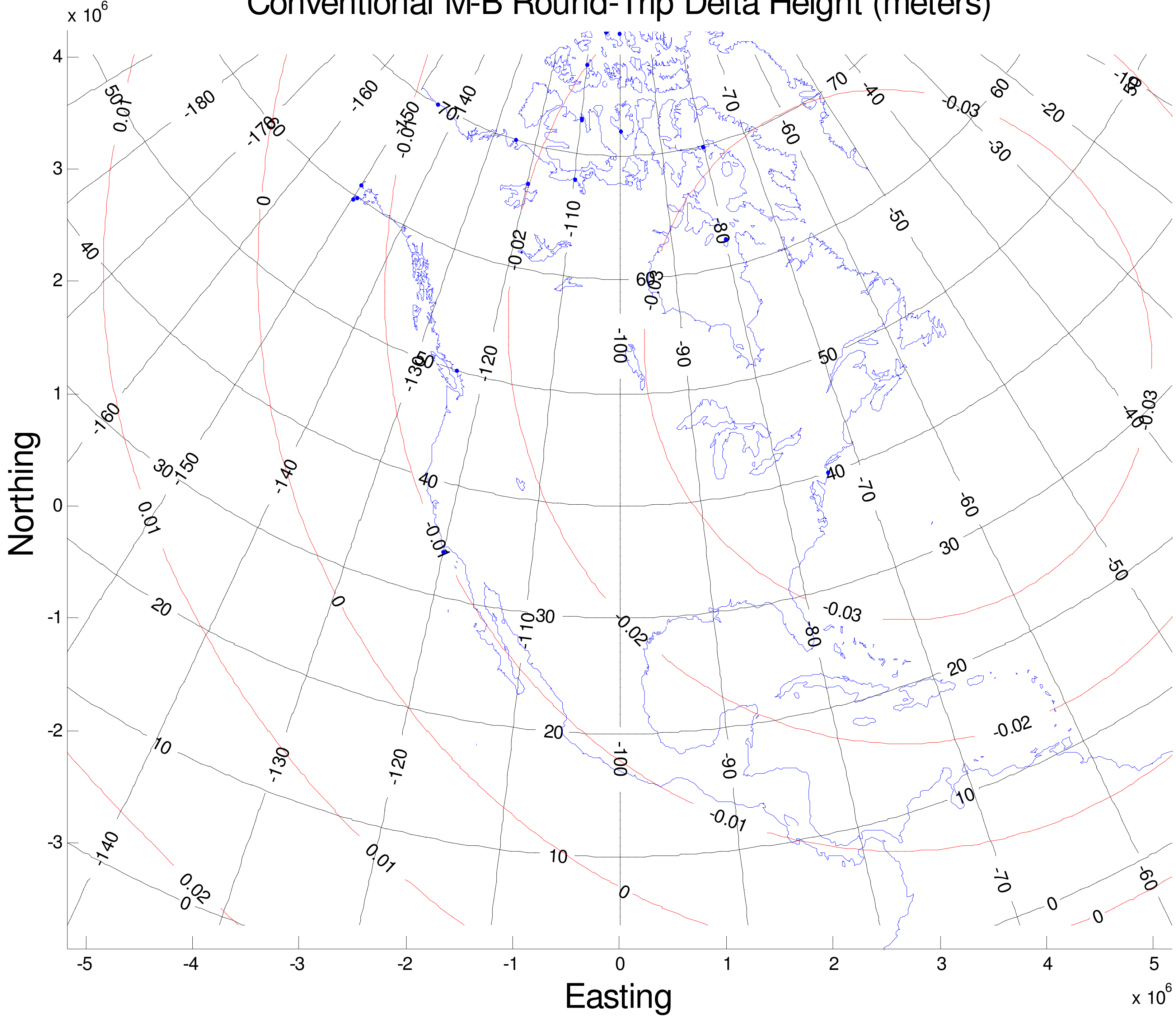
- The area chosen is all of North America
- The ellipsoids chosen for this exercise are Clarke 1866 and Bessel because of the range in size and inverse flattening
- The (large) translations are 1km each
- The (large) rotations are 10 arcseconds each
- The (large) scale change is 20ppm
- The area is presented in Lambert Conformal Conic in the next slide

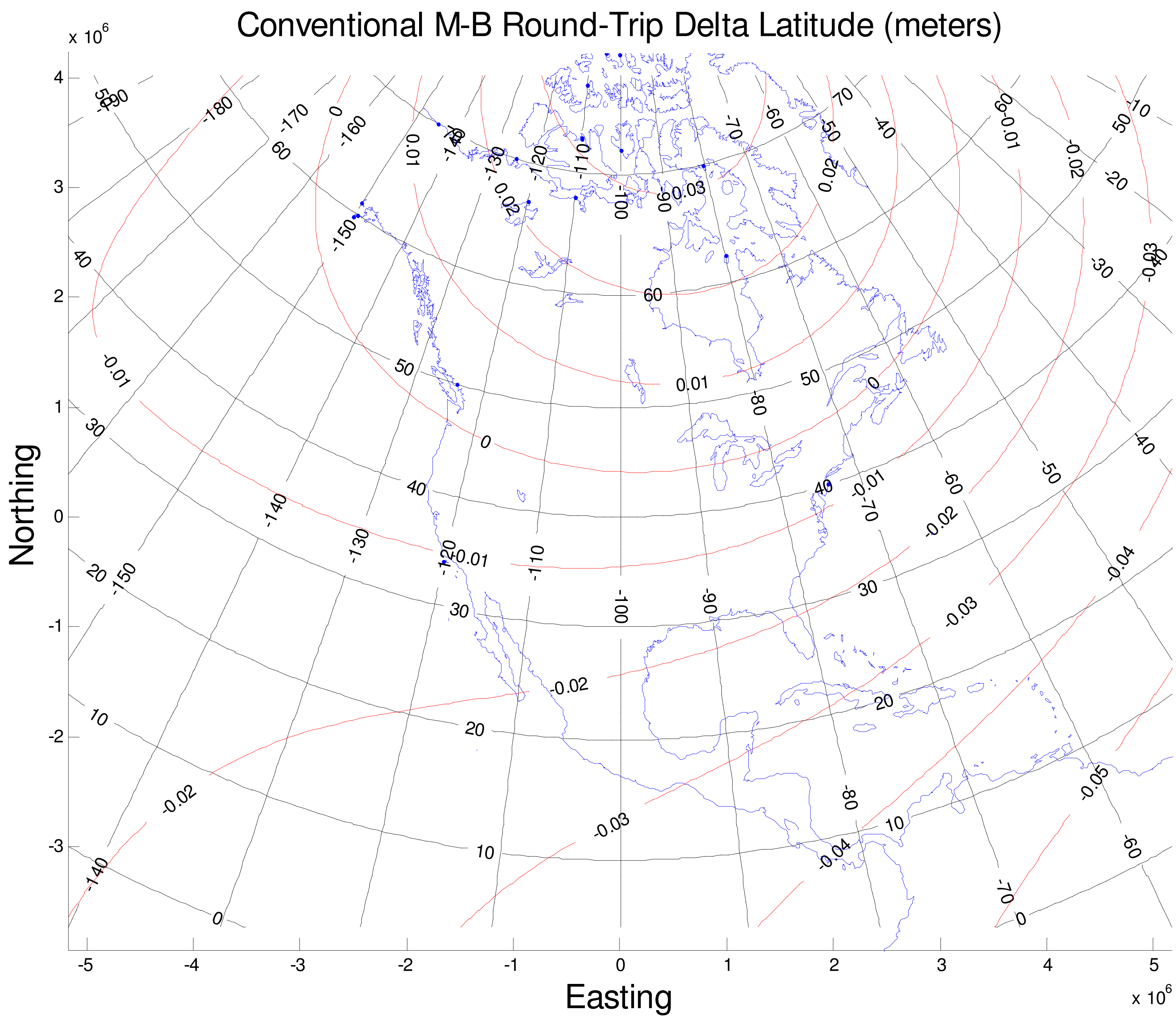

LCC Latitude/Longitude Graticule
Northing
Easting
x 10^6
x 10^6

Conventional Round Trip

- The next three slides show the round-trip mis-closures in height, latitude and longitude as contour plots of the LCC on North America
- Delta height is in meters, range is -3 => +2cm
- Delta latitude is converted from decimal degrees to meters, range is about -5 => +3cm
- Delta longitude values are also converted and range from about -4 => +4cm

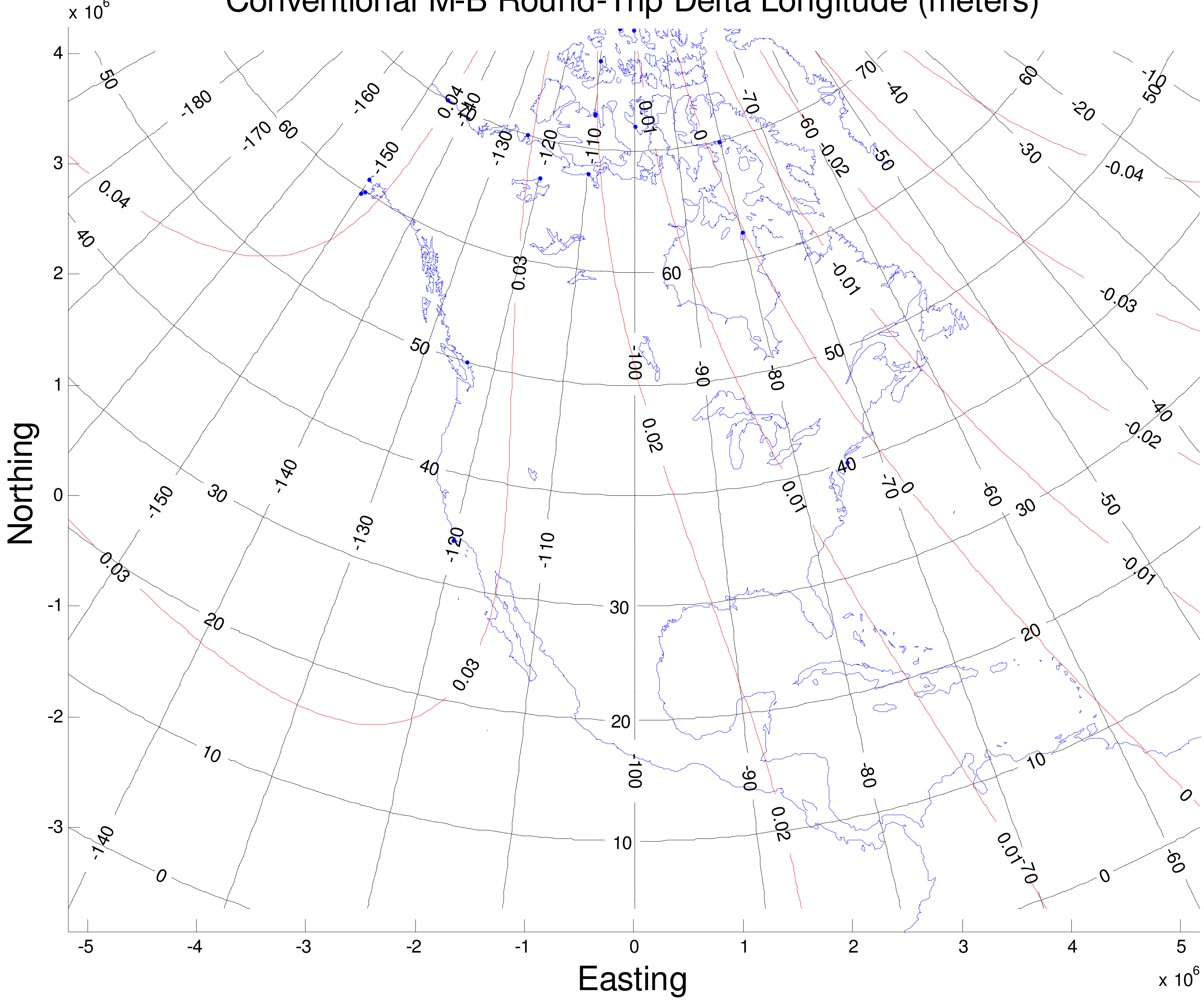
Conventional M-B Round-Trip Delta Height (meters)





Conventional M-B Round-Trip Delta Latitude (meters)
x 10^6
Northing
Easting
x 10^6

Conventional M-B Round-Trip Delta Longitude (meters)



Part I Conclusion

- A conventional reversal of the M-B (reversing the signs of the usual 7 parameters) results in round-trip mis-closures of 4-5cm at worst (large shift values over a large area)
- More normal usage of the M-B transformation (smaller parameters, smaller area) should result in mis-closures of 1-2cm
- (The strange character of these contours is due to the arbitrary choice of shift parameters)

Part II Contents

- A derivation of the non-linear, higher-order terms of the Dutch reversed M-B shift is given on the next slide. Those terms are:

$$R^2 \cdot (u' - u) - 2 \cdot R \cdot \Delta S \cdot (u - u') + \Delta S^2 \cdot (u' - u)$$

- This presentation offers a “worst-case” computation of the size of these higher-order terms over a very large area
- Non-reversibility is typically only a few centimeters

Forward Molodensky-Badekas

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} \Delta U \\ \Delta V \\ \Delta W \end{bmatrix} + \begin{bmatrix} 0 & \omega & -\psi \\ -\omega & 0 & \varepsilon \\ \psi & -\varepsilon & 0 \end{bmatrix} \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i + \Delta S \begin{bmatrix} U - U' \\ V - V' \\ W - W' \end{bmatrix}_i$$

$$x = u + \Delta u + R \cdot (u - u') + \Delta S \cdot (u - u')$$

Dutch Reverse M-B

$$u_r = x - \Delta u - R \cdot (x - u' - \Delta u) - \Delta S \cdot (x - u' - \Delta u)$$

$$u_r = u + [R^2 \cdot (u' - u) - 2 \cdot R \cdot \Delta S \cdot (u - u') + \Delta S^2 \cdot (u' - u)]$$

Dutch M-B Reverse

- Dutch reverse the M-B means this:
  - Changing the order of the two ellipsoids
  - Negating the signs of 7 of the 10 parameters, viz., ΔX, ΔY, ΔZ, rX, rY, rZ, and ΔScale
  - Adding the translations to the geocentric coordinates for the rotation center
- New rotation center coordinates are used, i.e., all 10 of 10 parameters change (in addition to reversing the order of the ellipsoids)

Dutch Forward and Reverse M-B

```
% Forward M-B Transformation
% Clarke 1866
A_in =     6378206.4;
RF_in =  294.9786982;
dX =            1000;
dY =           -1000;
dZ =            1000;
rotXsec =        -10;
rotYsec =         10;
rotZsec =        -10;
dS =       20*10^-6;
% 40N/100W/0Hgt on Clarke
X0 =     -849632.077;
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Z0 =     4077787.743;
% Bessel
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% Reverse M-B Transformation
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dX =           -1000;
dY =            1000;
dZ =           -1000;
rotXsec =         10;
rotYsec =        -10;
rotZsec =         10;
dS =      -20*10^-6;
% 40N/100W/0Hgt on Bessel
X0 =     -850632.077;
Y0 =    -4817502.951;
Z0 =     4076787.743;
% Clarke 1866
A_out =    6378206.4;
RF_out = 294.9786982;
```

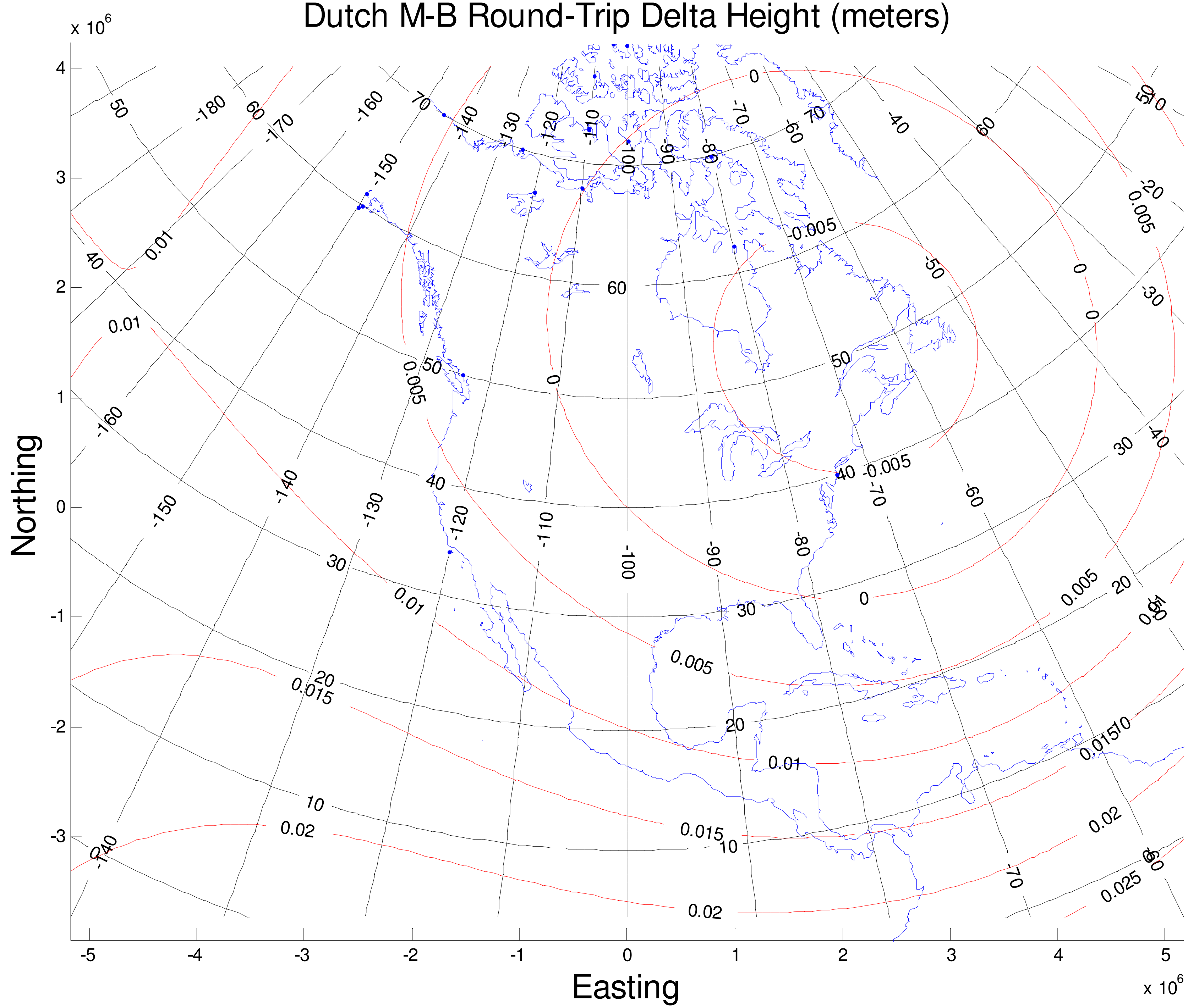
Transformation Parameters

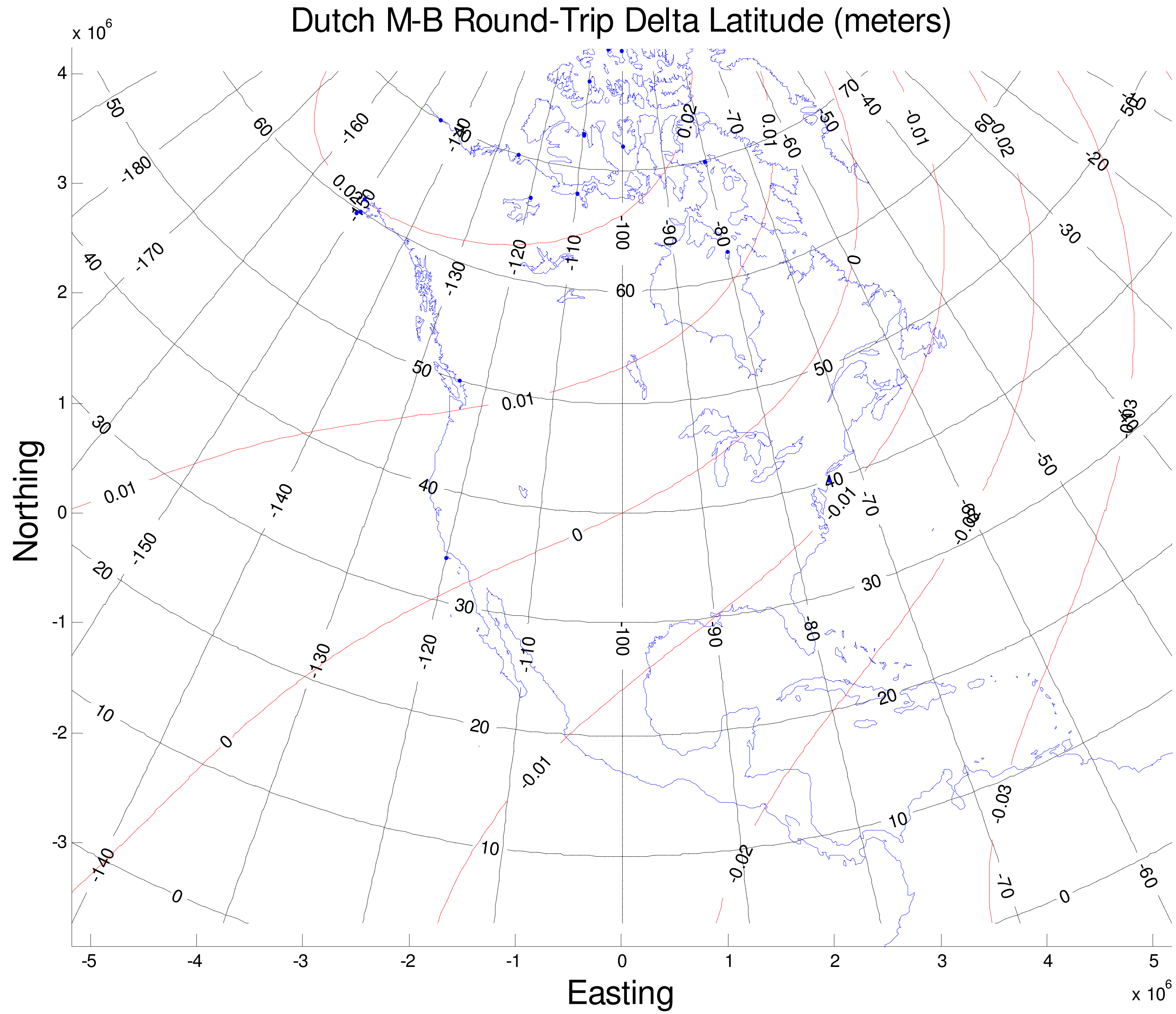
- The area chosen is all of North America
- The ellipsoids chosen for this exercise are Clarke 1866 and Bessel because of the range in size and inverse flattening
- The (large) translations are 1km each
- The (large) rotations are 10 arcseconds each
- The (large) scale change is 20ppm

Dutch Round Trip

- The next three slides show the round-trip mis-closures in height, latitude and longitude as contour plots of the LCC on North America
- Delta height is in meters, range -0.5 => +2cm
- Delta latitude is converted from decimal degrees to meters, range is about -3 => +2cm
- Delta longitude values are also converted and range from about -3 => +2cm

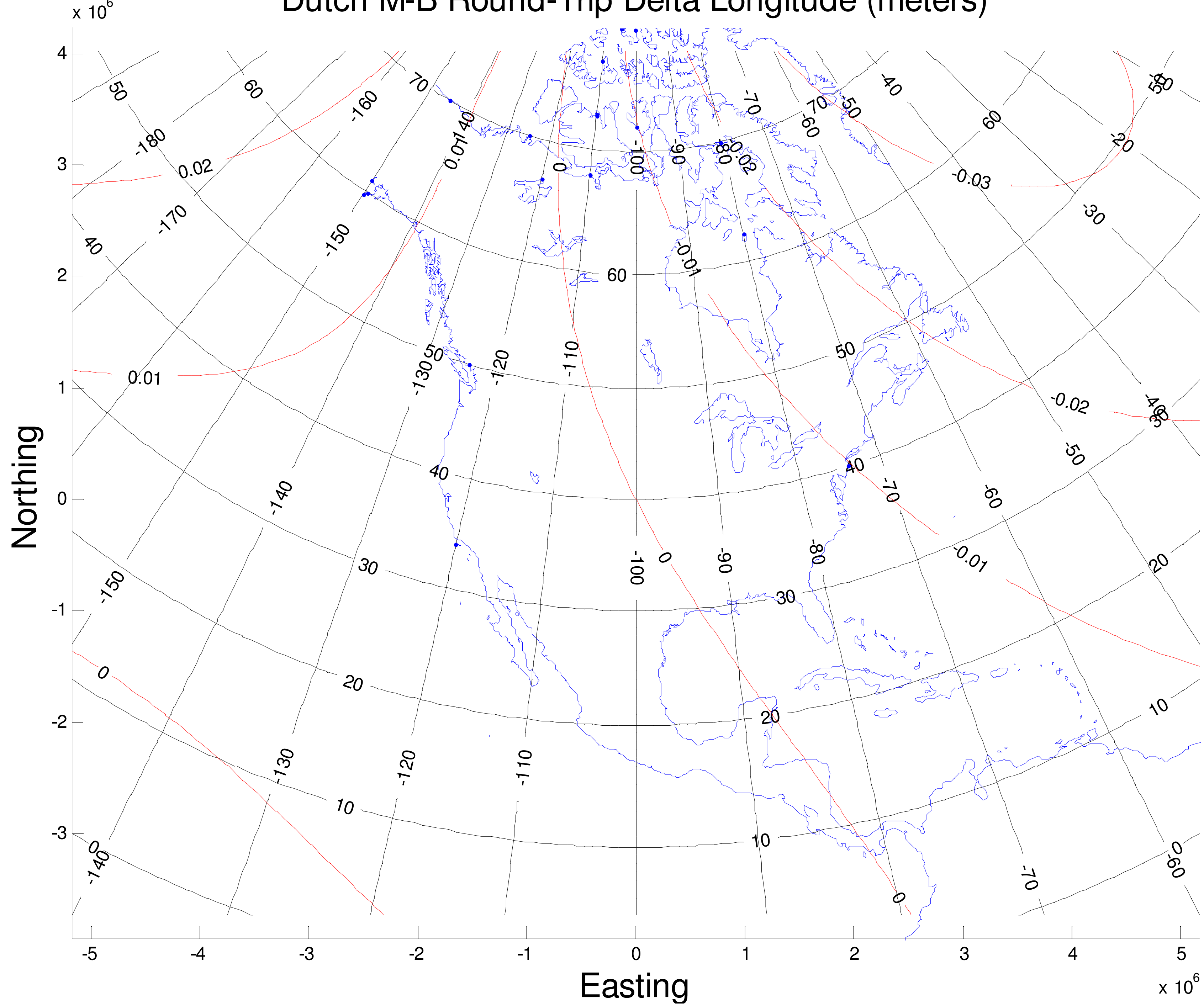
Dutch M-B Round-Trip Delta Height (meters)





Dutch M-B Round-Trip Delta Latitude (meters)
x 10^6
Northing
Easting
x 10^6

Dutch M-B Round-Trip Delta Longitude (meters)



Part II Conclusion

- A Dutch reversal of the M-B results in round-trip mis-closures of 2-3cm at worst
- More normal usage of the M-B transformation (smaller parameters, smaller area) should result in mis-closures of about 1cm
- Dutch method is about 40% more reversible than the conventional method